



# A SYMMETRIC SPATIAL VERIFICATION METHOD FOR SEA ICE EDGES

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November 13 @ 15:00 UTC, 2020

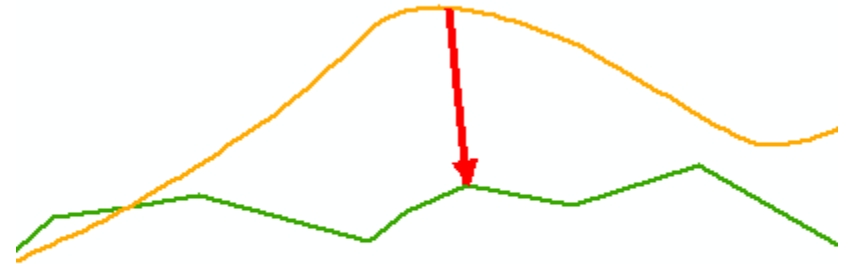


# ICE EDGE VERIFICATION METHODS

AREA



DISTANCE

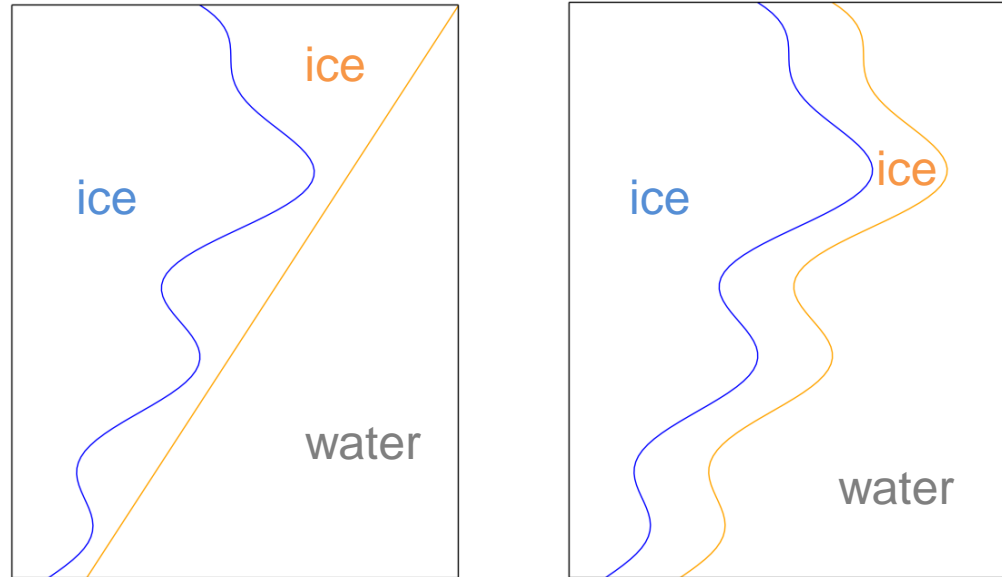


Goessling, H. F., Tietsche, S., Day, J. J., Hawkins, E., and Jung, T. (2016), Predictability of the Arctic sea ice edge, *Geophys. Res. Lett.*, 43, 1642– 1650, doi:[10.1002/2015GL067232](https://doi.org/10.1002/2015GL067232).

Do they overlap?

Near or far?

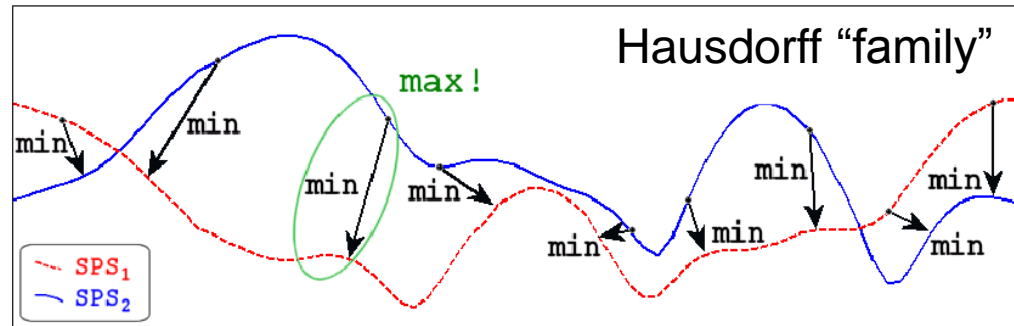
# WHY USE A DISTANCE METRIC?



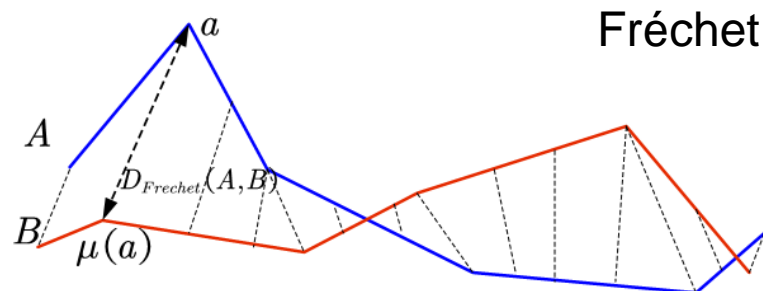
Similarity measure: it answers how similar two lines are

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# DISTANCE METHODS



Kaspar, D.. "Application of Directional Antennas in RF-Based Indoor Localization Systems." (2005).



Guo, Ning & Ma, Mengyu & Xiong, Wei & Chen, Luo & Jing, Ning. (2017). An Efficient Query Algorithm for Trajectory Similarity Based on Fréchet Distance Threshold. ISPRS International Journal of Geo-Information. 6. 326. 10.3390/ijgi6110326.

The Fréchet distance is more computationally complex;  
most applications use Hausdorff in practice.

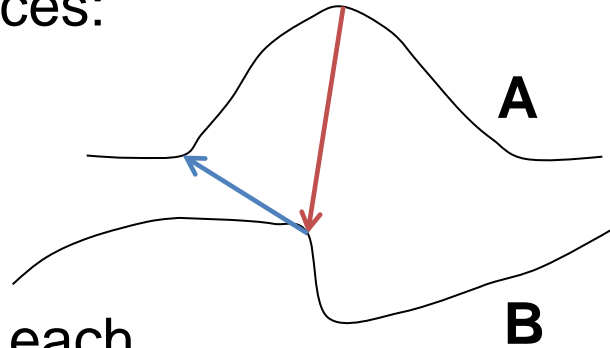
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# DISTANCES WITHIN THE HAUSDORFF FAMILY

DUBUISSON AND JAIN (1994)

Forward distances:

$$d(a, B)_{a \in A}$$



distances from each grid-point  $a$  belonging to the set A, to the set B

Backward distances:

$$d(b, A)_{b \in B}$$

distances from each grid-point  $b$  belonging to the set B, to the set A

The metric is made symmetric by taking the maximum:

$$Haus(A, B) = \max\{\max_{a \in A} d(a, B); \max_{b \in B} d(b, A)\}$$

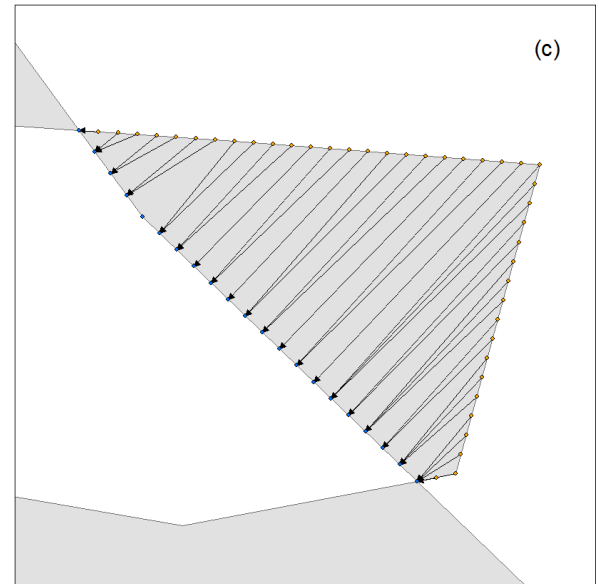
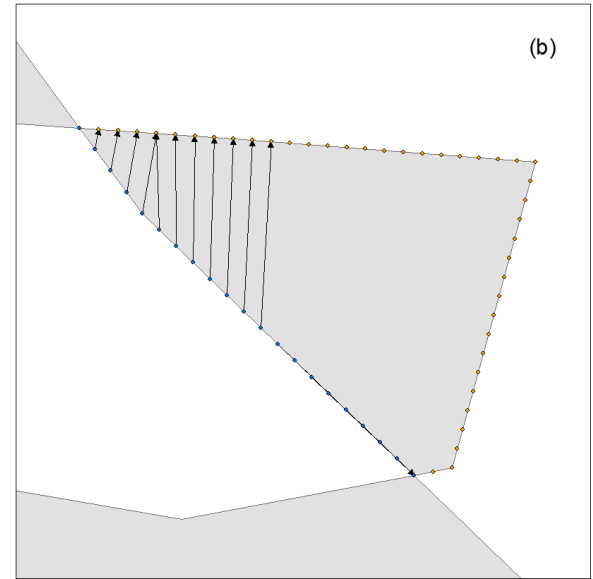
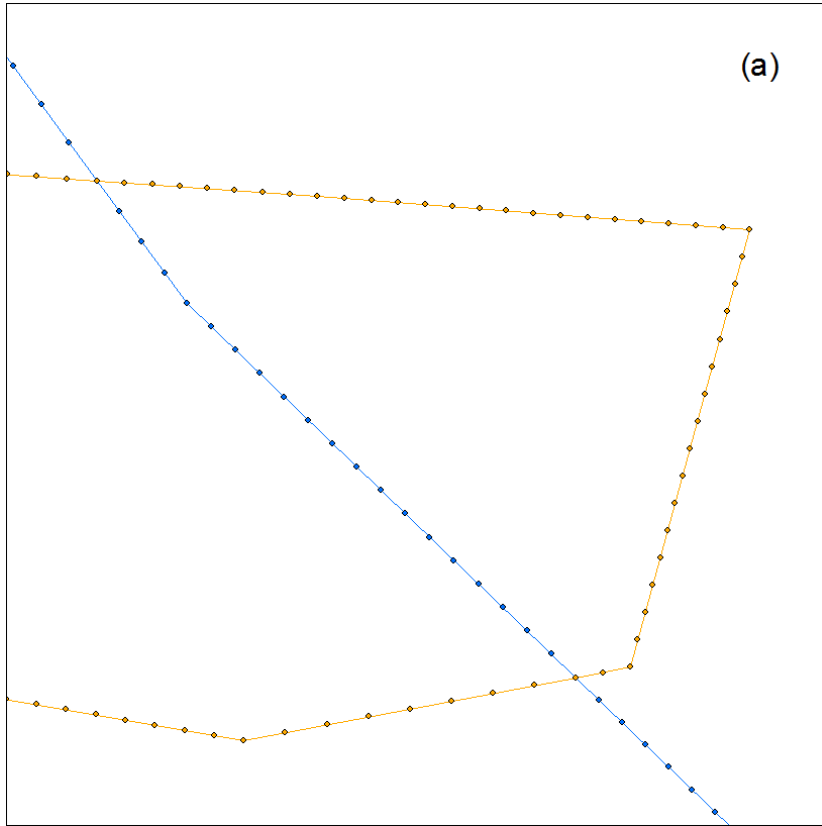
$$PartHaus(A, B) = \max\{q_{0.50} d(a, B)_{a \in A}; q_{0.50} d(b, A)_{b \in B}\}$$

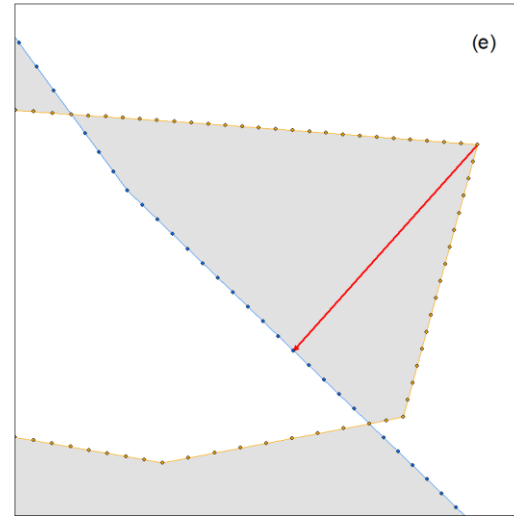
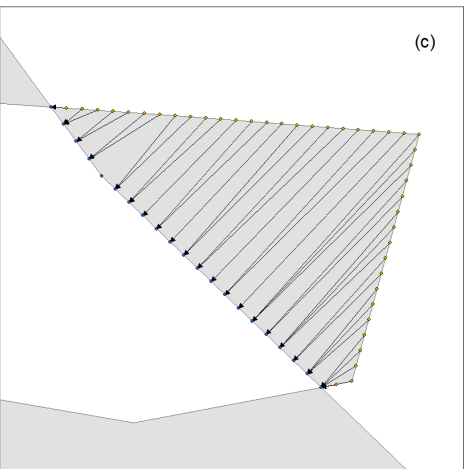
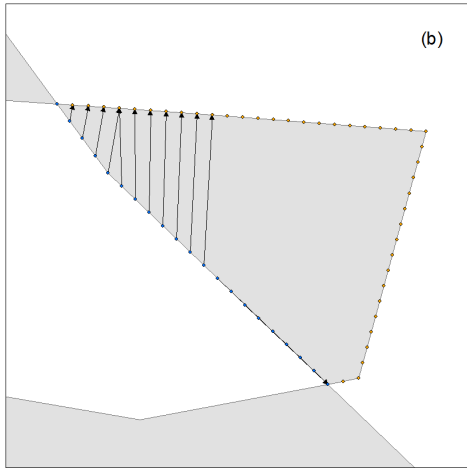
$$ModHaus(A, B) = \max\{\text{mean}_{a \in A} d(a, B); \text{mean}_{b \in B} d(b, A)\}$$

# DO WE WANT A METRIC?

**Definition:** a metric  $M$  between two sets of pixels  $A$  and  $B$  satisfies:

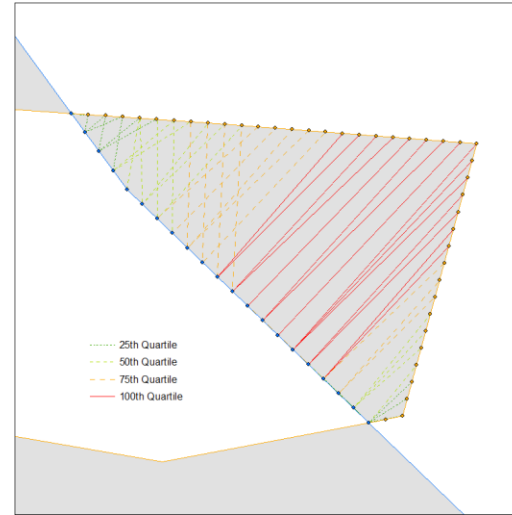
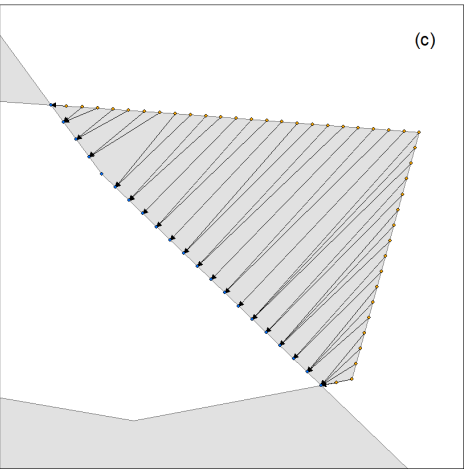
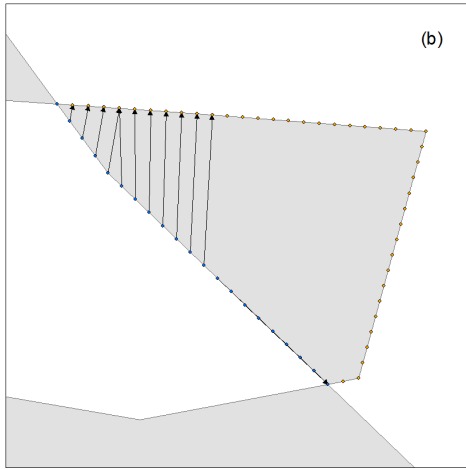
1. **Positivity:**  $M(A,B) \geq 0$
  2. **Separation:**  $M(A,B) = 0$  if and only if  $A = B$
  3. **Symmetry:**  $M(A,B) = M(B,A)$
  4. **Triangle Inequality:**  $M(A,C) + M(C,B) \geq M(A,B)$
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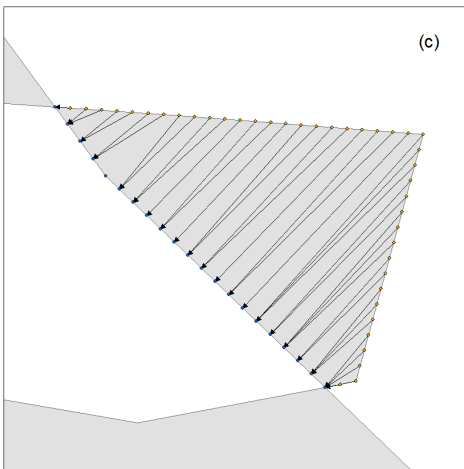
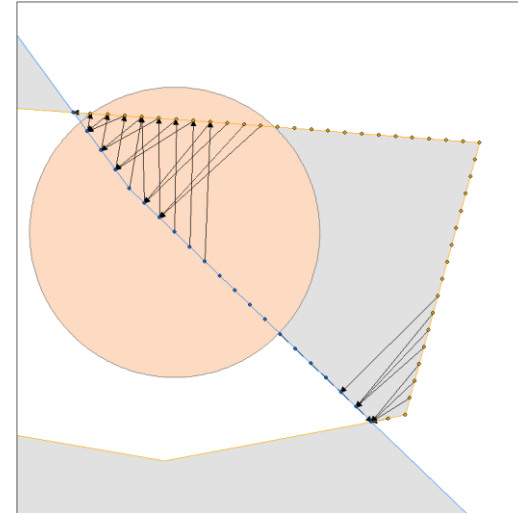
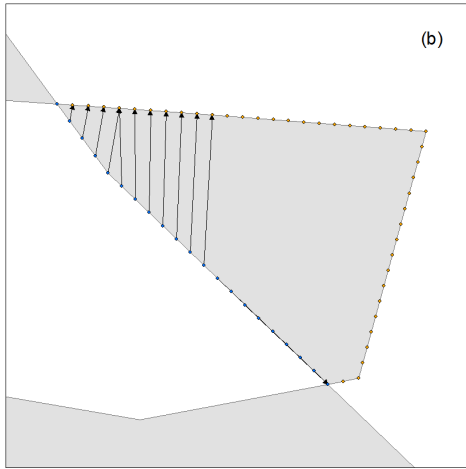


$$Haus(A,B) = \max\{\max_{a \in A} d(a, B); \max_{b \in B} d(b, A)\}$$





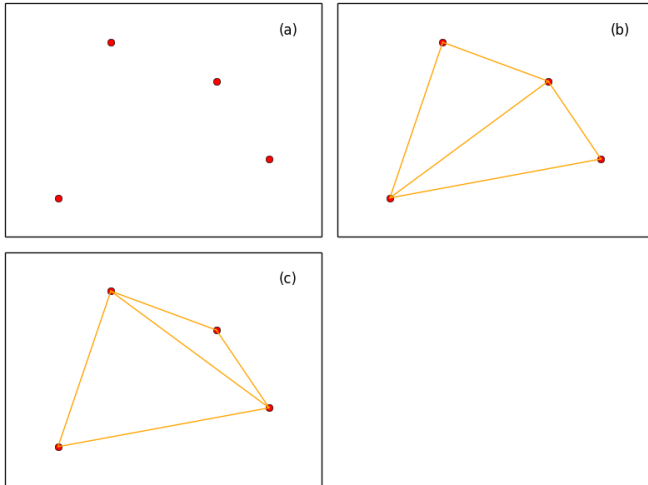
$$PartHaus(A,B) = \max\{q_{0.50} d(a, B)_{a \in A}; q_{0.50} d(b, A)_{b \in B}\}$$



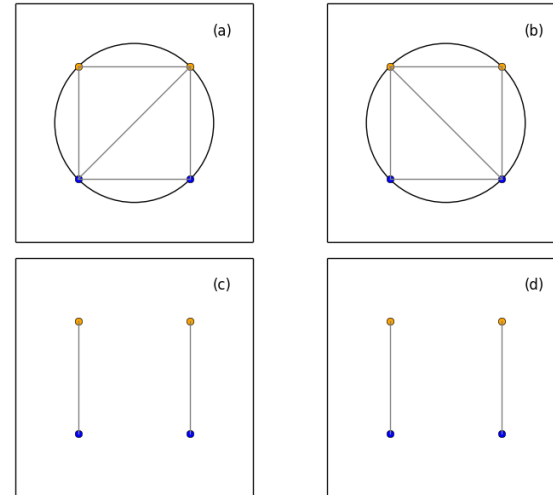
$$\text{ModHaus}(A,B) = \max\{\text{mean}_{a \in A} d(a, B) ; \text{mean}_{b \in B} d(b, A)\}$$

# ALGORITHM

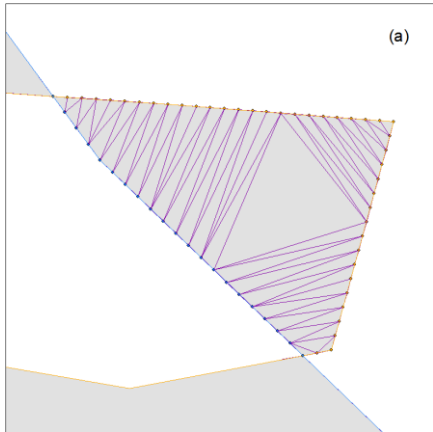
Step 1: Triangulate.



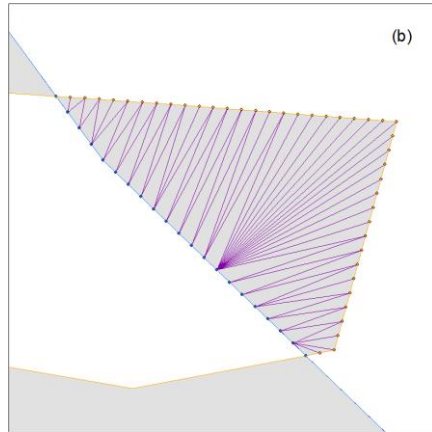
Step 2: Make the solution unique.



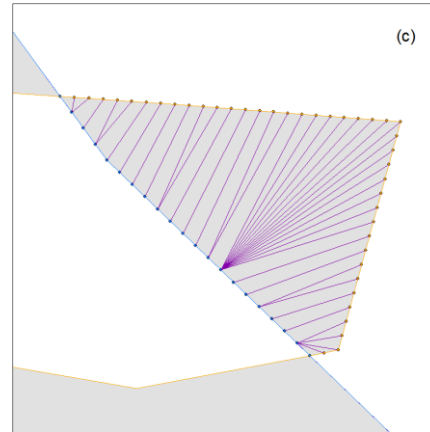
Step 1



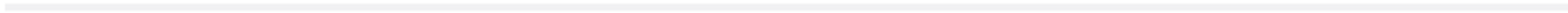
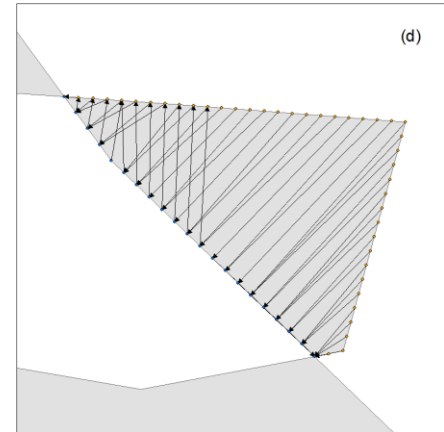
Step 2



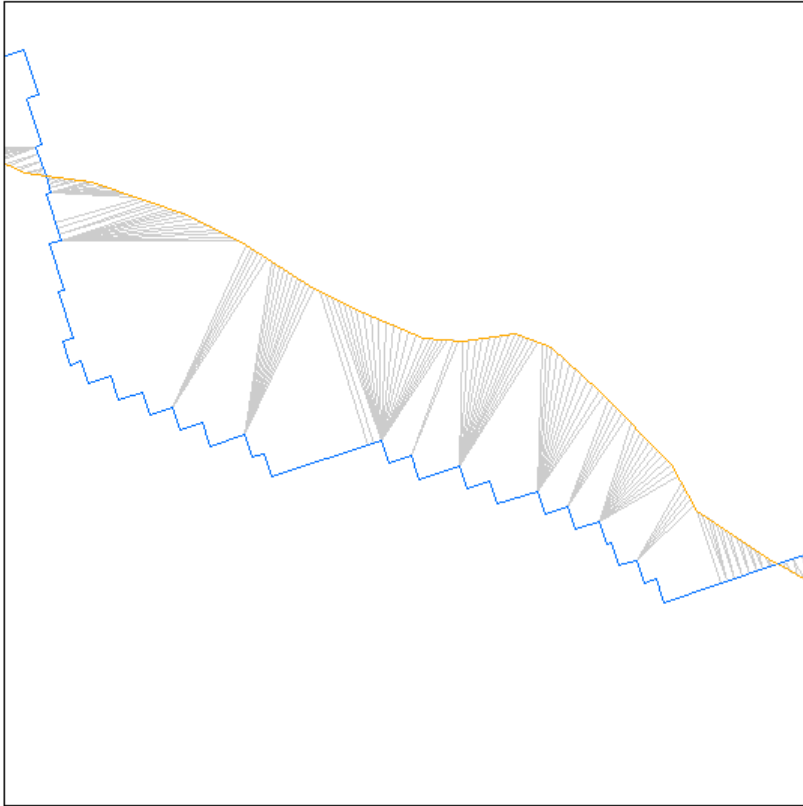
Final result



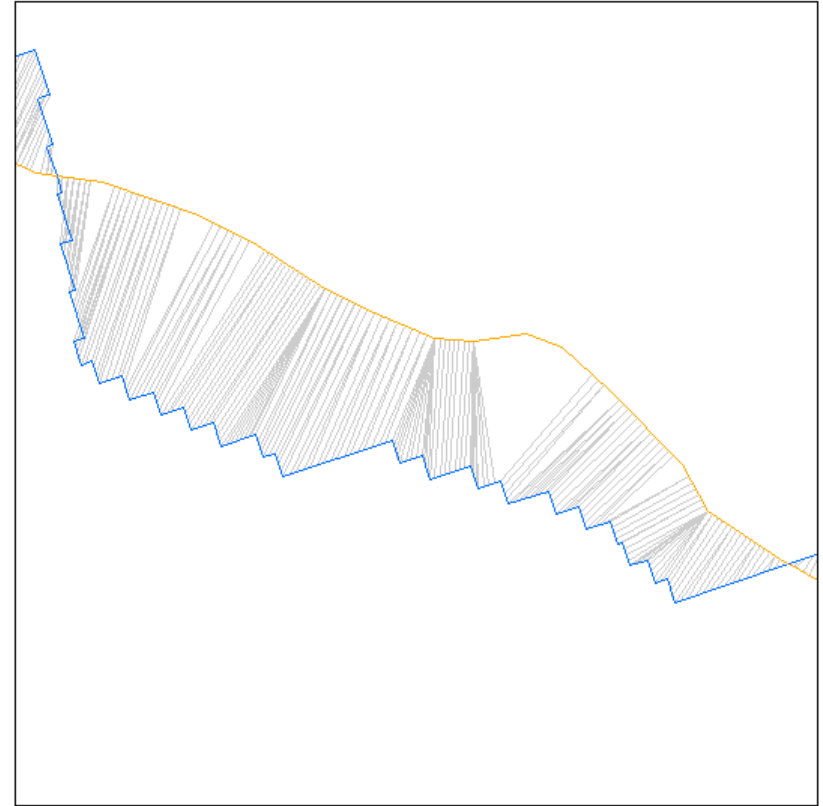
Forward/  
backward  
passes



Forward



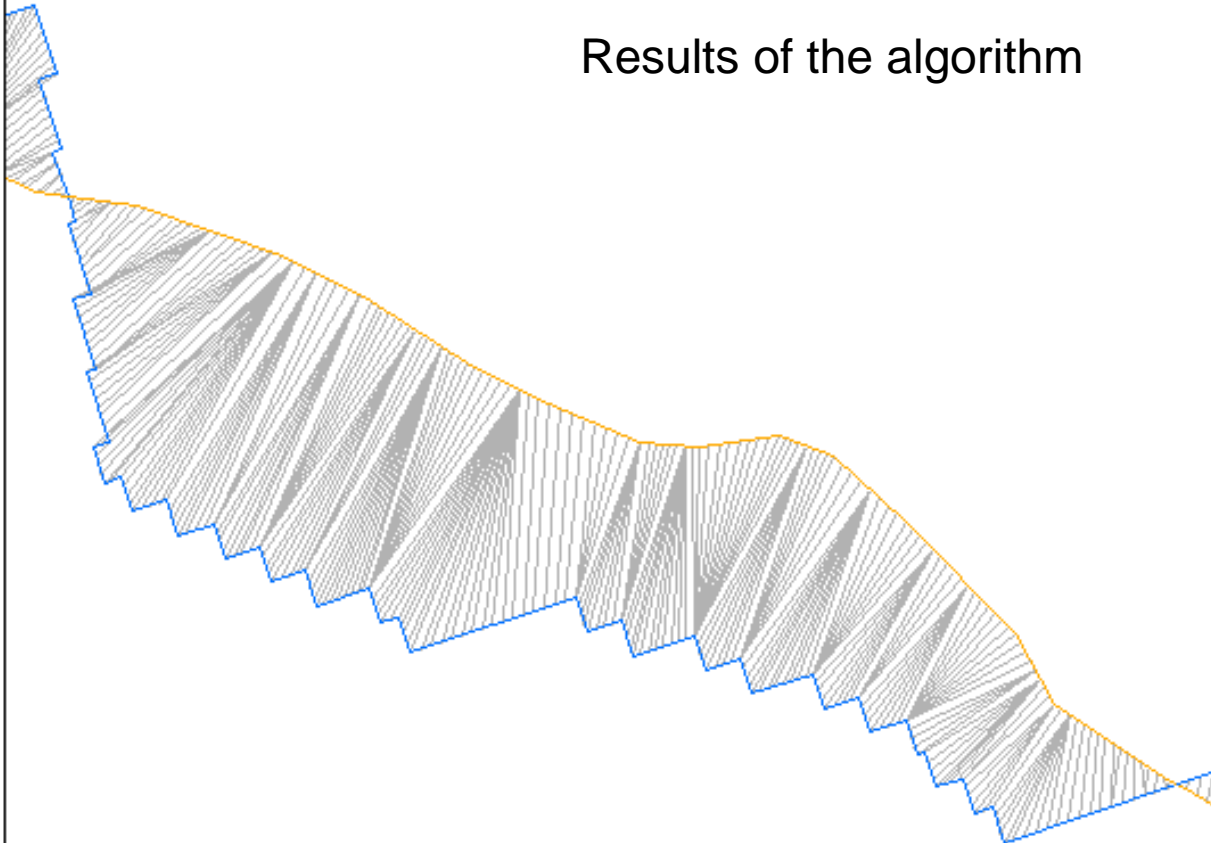
Backward



The limitation is that it does not account for full sequencing of points.

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## Results of the algorithm



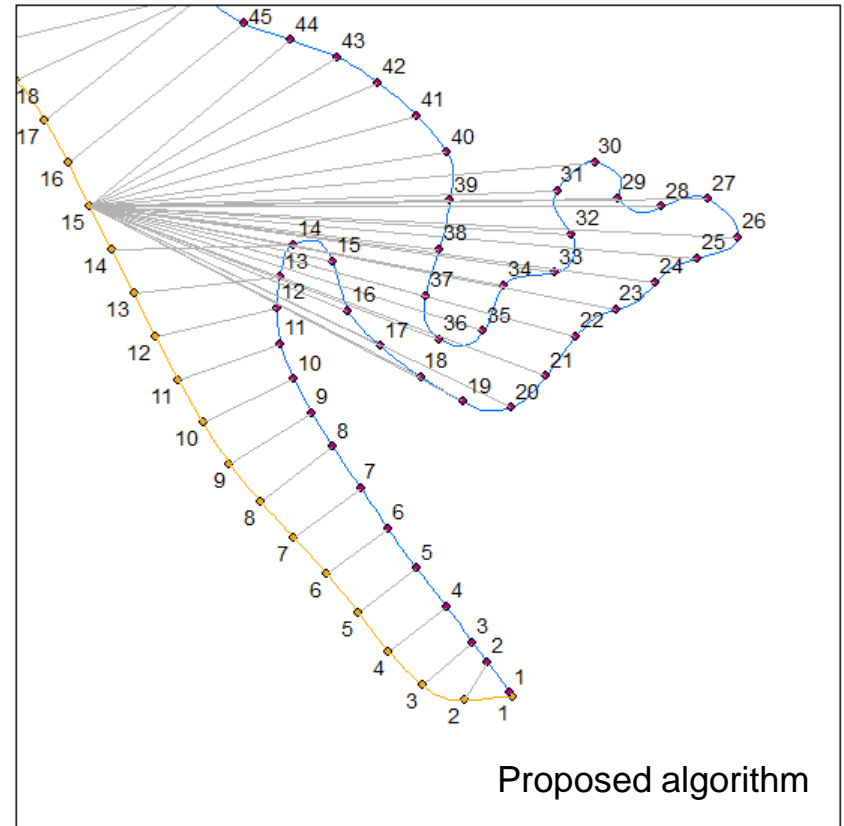
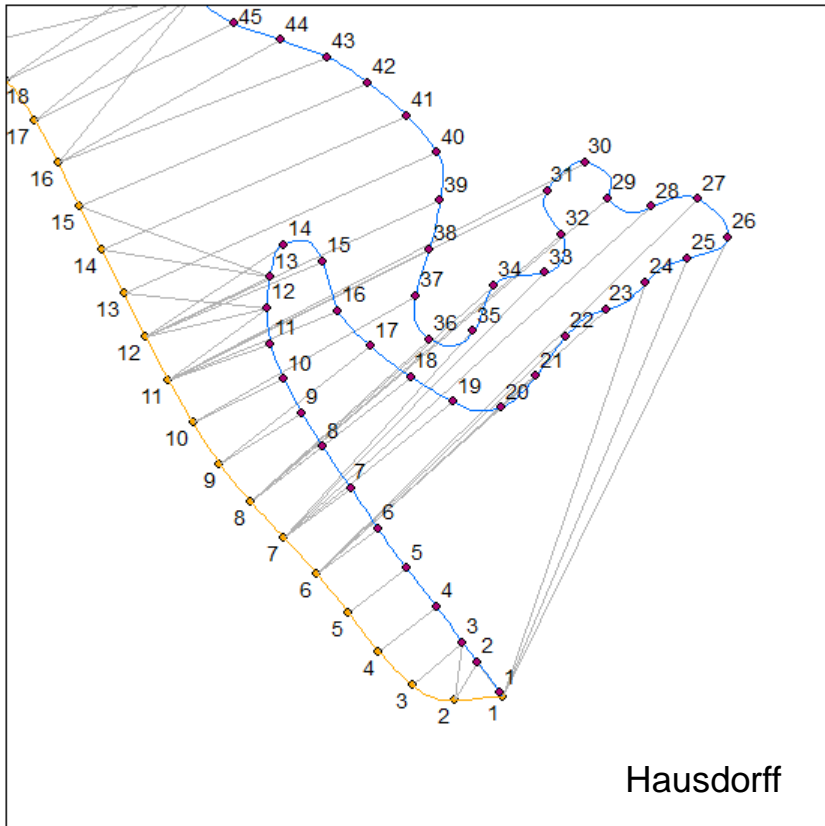
$$d(x, y) \geq 0 \text{ for all } x, y \in X$$

$$d(x, y) = 0 \Leftrightarrow x = y$$

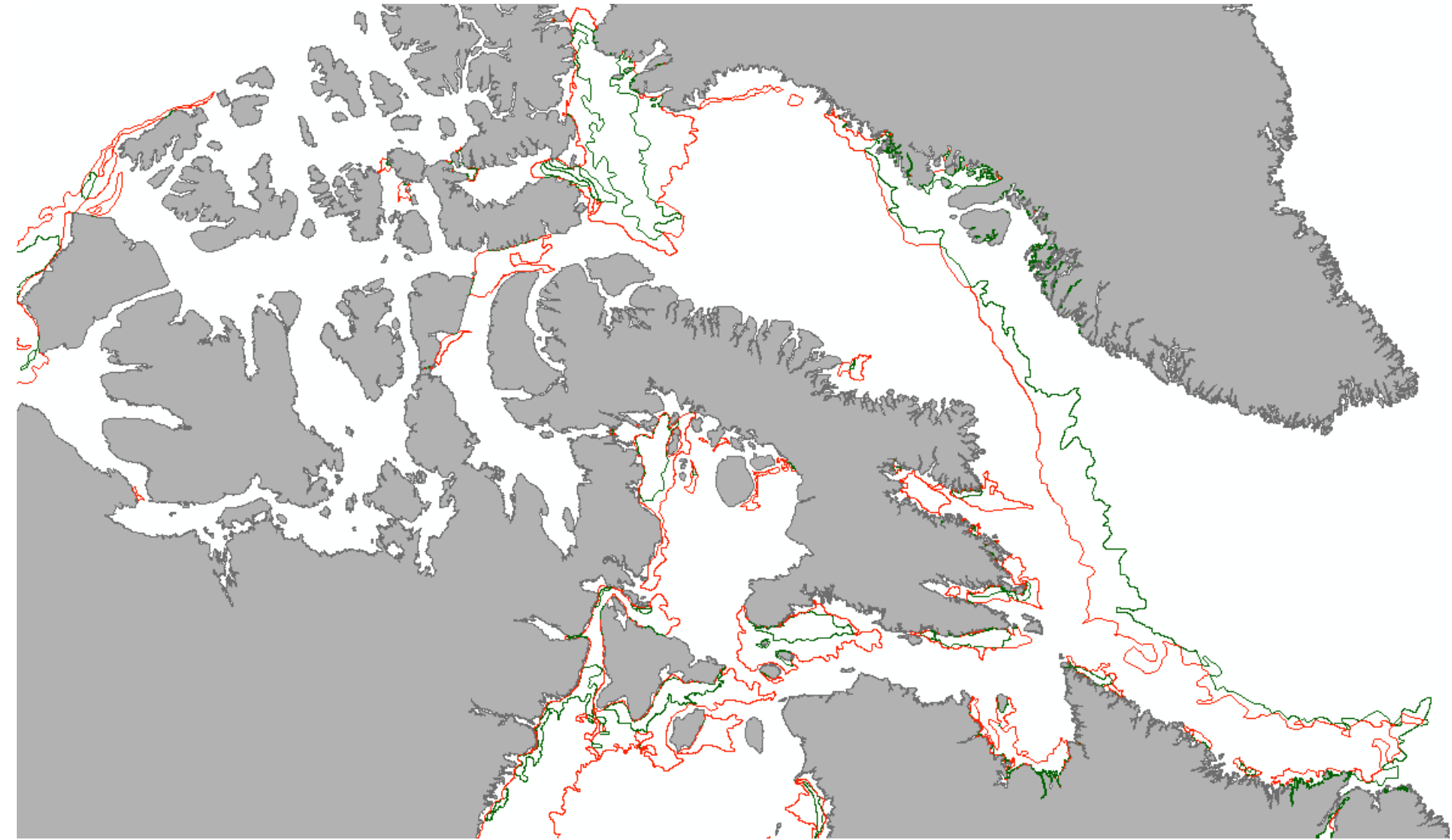
$$d(x, y) = d(y, x)$$

$$d(x, y) \leq d(x, z) + d(z, y)$$

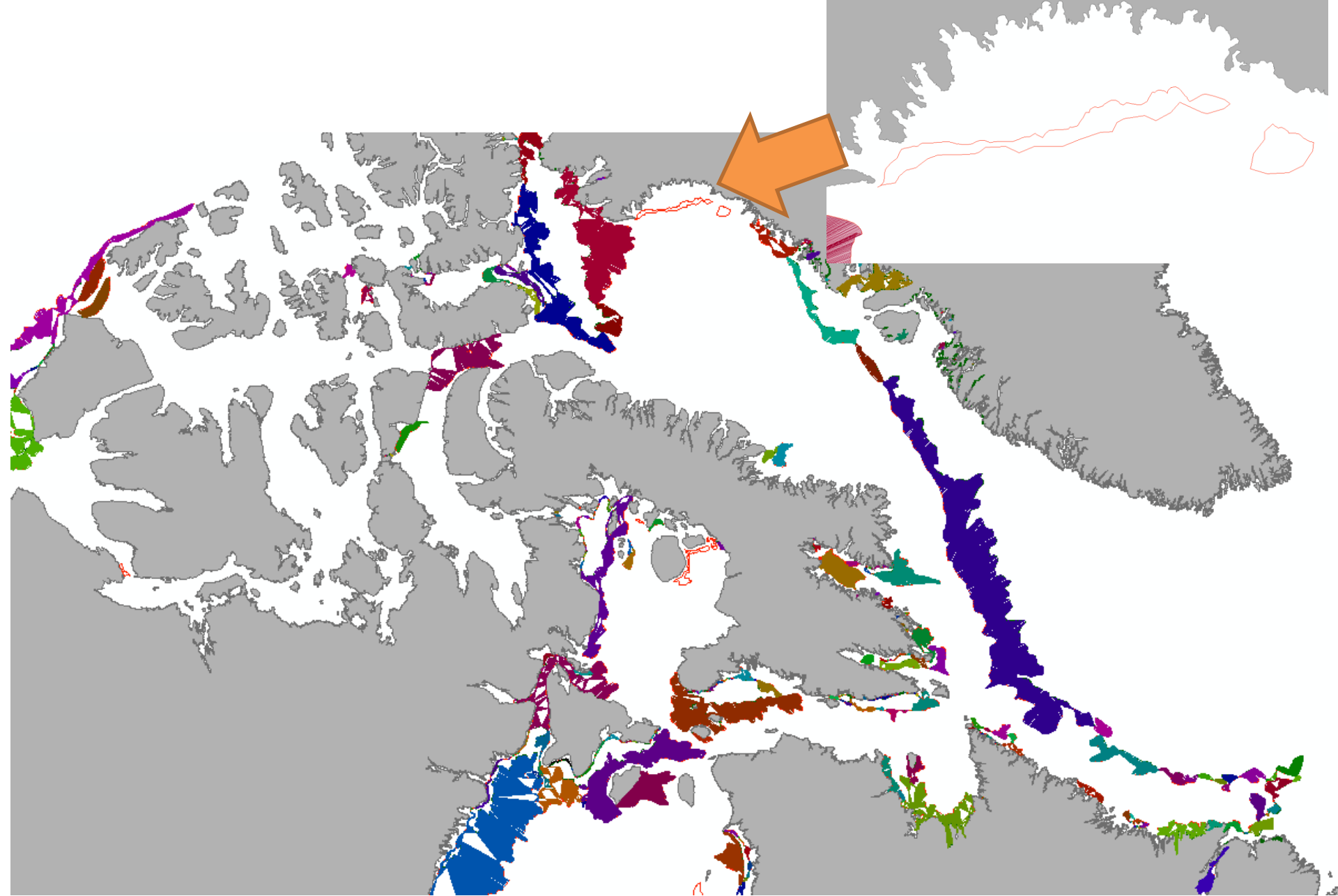
Hausdorff—points are skipped so it is out of sequence.



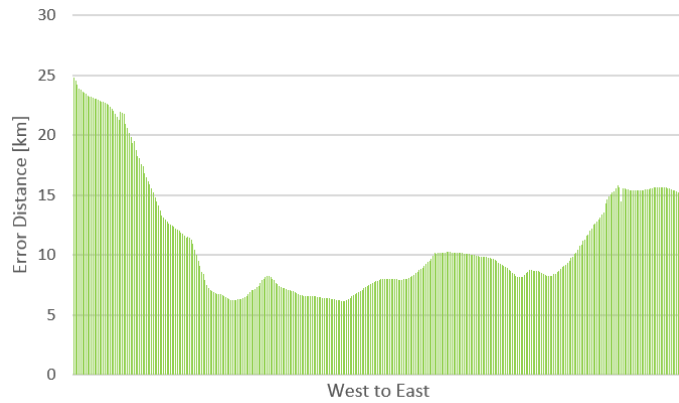
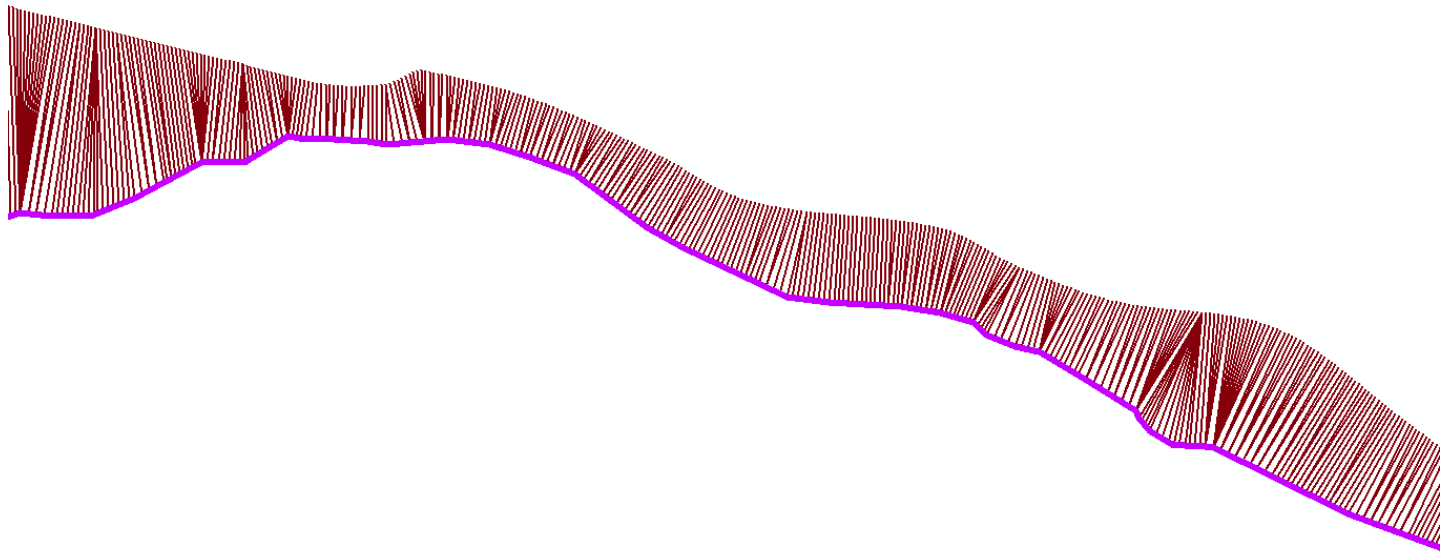
What about new ice growth, when there is no edge to compare against?



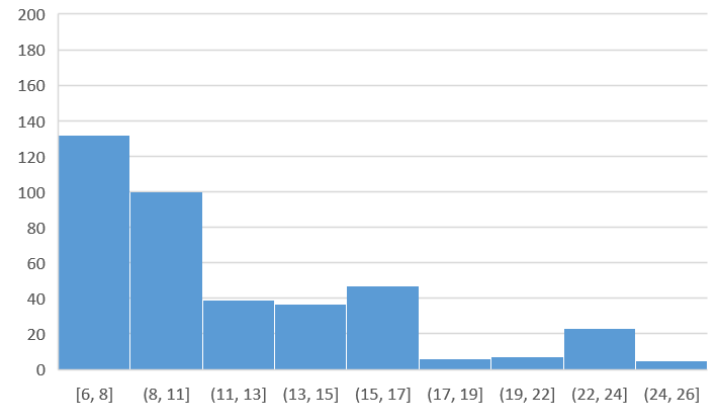




New ice growth is not considered. If there is no linear feature to compare against, then the algorithm ignores it.

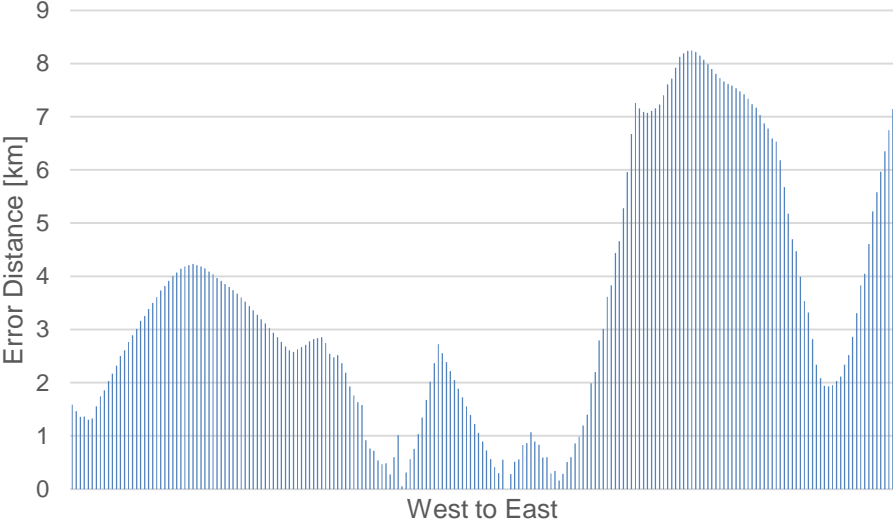
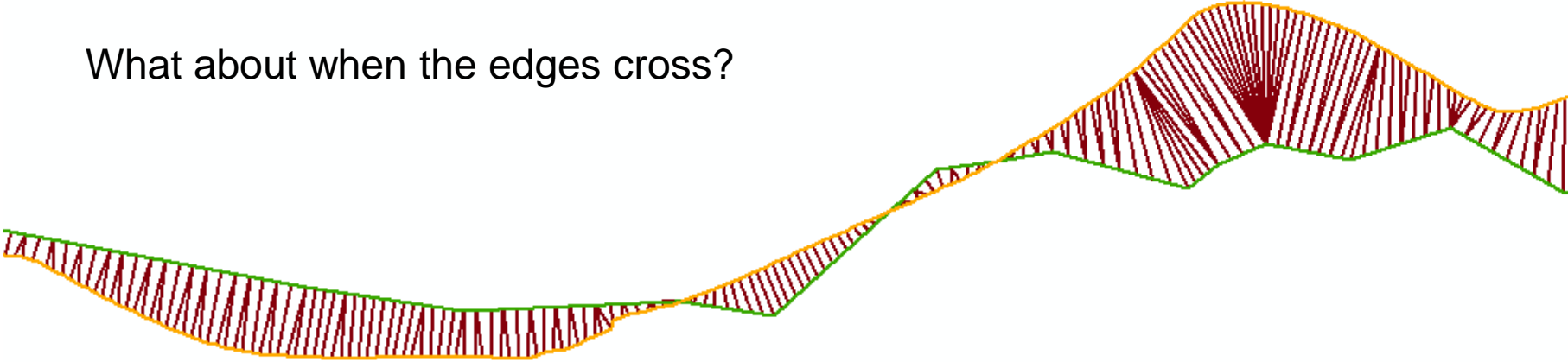


Sequence of distances shown in order

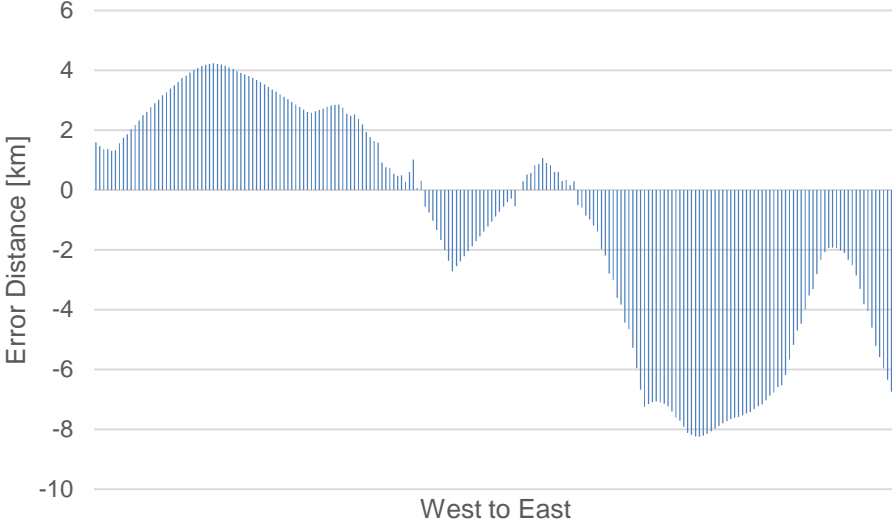


Sequence of distances shown as histogram

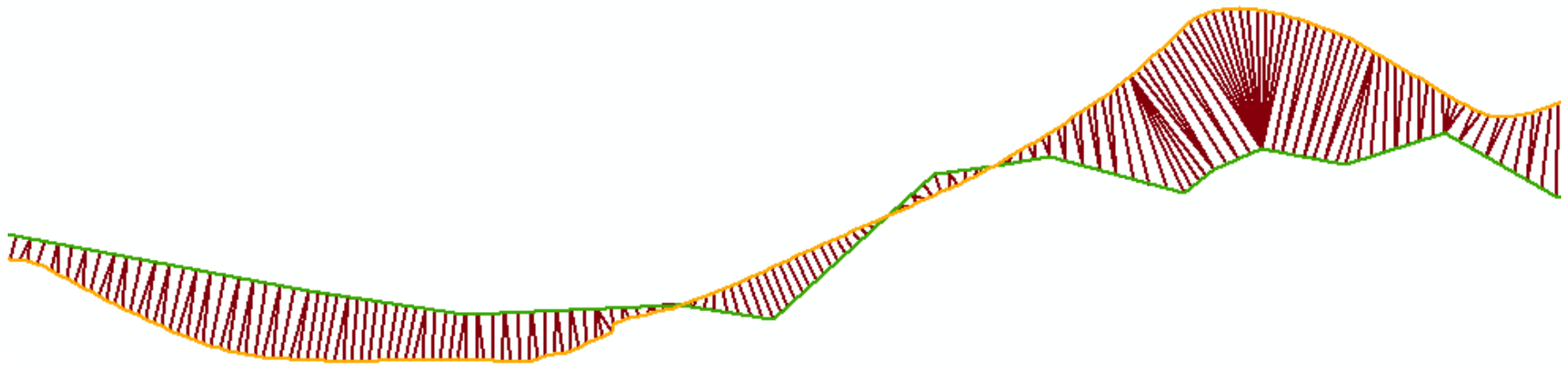
What about when the edges cross?



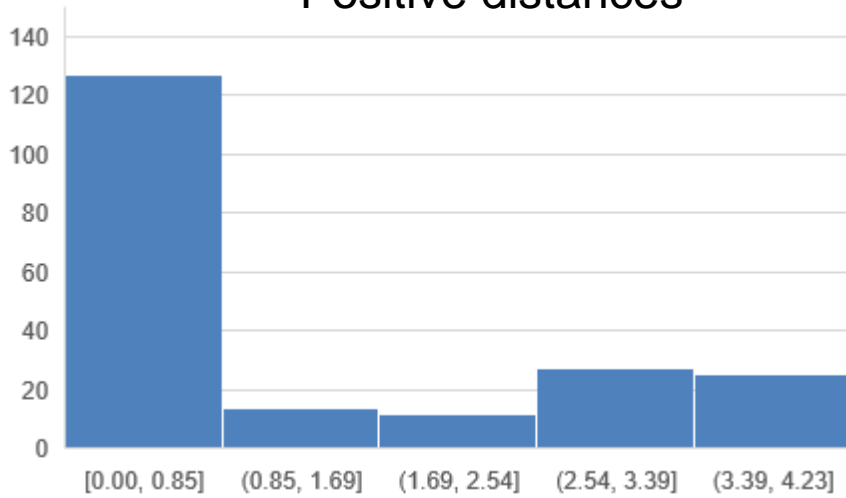
Absolute distances



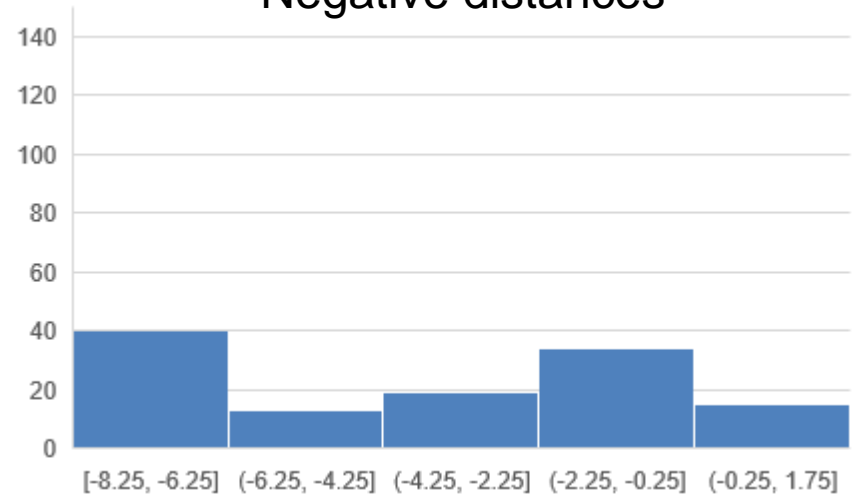
Signed distances



Positive distances

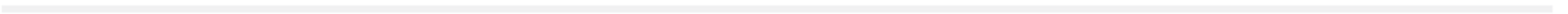


Negative distances



# NEXT STEPS?

- Try the algorithm on some YOPP datasets





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Climate Change Canada

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Canada 