A mutual information theory-based score for assessing the uncertainty in multi-category precipitation forecasts

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Outline

01 Motivation

02 Methodology

03 Data and study area

04 Results

05 Conclusions

1 Motivation

The motivation to verify the multi-category precipitation forecasts

- The **precipitation forecasts** from numerical weather prediction have a variety of potential uses in flood forecasting and reservoir operation, but suffer from relatively poor performance due to the **uncertainty** of the hydrometeorological system.

Application of multi-category precipitation forecasts on reservoir operation

Forecasts	Decisions	Results
No rain, Light rain	Store more water to avoid water shortage	No rain or Light rain occurs (successs)
		Heavy rain or Rainstorm occurs (failure) *Risk of dam break!*
• • •		

- **Verification of multi-category precipitation forecasts is necessary for reservoir operation**.

2 Methodology

Related research on forecast verification using mutual information (MI)

Weijs et al. (2010) presented the DS score that can be used for evaluating **probabilistic forecasts** of multi-category events.
The DS score can be decomposed into 3 components like the BS score. **Resolution** is the **MI** of forecasts and observations.

$DS = REL_{DS} - RES_{DS} + UNC_{DS}$ $\quad$ $RES_{DS} = I(F,O)$

Hughes (2012) introduced the application of **MI** on the forecast verification in the field of **epidemiology** and decomposed MI into **each category**.

The **difference** between this research with the two papers is that MI is used for evaluating **deterministic** multi-category **precipitation forecasts**.

2 Methodology

A proposed score for assessing the comprehensive uncertainty of all categories of precipitation forecasts

Proposed score	$NMI = \frac{H(X) - H(X\|Y)}{H(X)} = \frac{I(X,Y)}{H(X)}$
The definition of each variable	X : Observed precipitation ; Y : Forecast precipitation; $H(X)$: The entropy of X ; represents the uncertainty in X ; $H(X\|Y)$: The conditional entropy of X given Y; represents the the amount of uncertainty remaining about X after Y is known; $I(X,Y)$: The mutual information of X and Y; represents the amount of uncertainty eliminated about X through observing Y; **NMI: The normalized mutual information; represents the ratio of uncertainty eliminated about X through observing Y ; The bigger the better.**

Entropy is a common tool to measure uncertainty.

2 Methodology

A proposed score for assessing the uncertainty of a certain category of precipitation forecasts

Decomposition of NMI:

$$NMI = \frac{I(X,Y)}{H(X)} = \sum_{Li=1}^{K} p(y_{Li}) \times NMI_{Li}$$

$$NMI_{Li} = \frac{H(X) - H(X|y_{Li})}{H(X)}$$

y_{Li}: The Li category forecast precipitation

$p(y_{Li})$: The probability of the occurrence of y_{Li}

K : The total number of categories of precipitation forecasts

$H(X|y_{Li})$: The entropy of X given y_{Li}; The amount of uncertainty remaining about X after receiving the Li category precipitation forecast information

The proposed score NMI_{Li}: ratio of uncertainty eliminated about X after receiving the Li category precipitation forecast information; The bigger the better.

3 Data and study area

Hunhe basin

- The **contradiction** between **flood control** and **water supply** is prominent in the Hunhe River basin.

- The **reservoir operation considering** forecast information such as **precipitation forecasts** is an effective way to **alleviate this problem**. However the **uncertainty** of forecast information may bring **risks**.

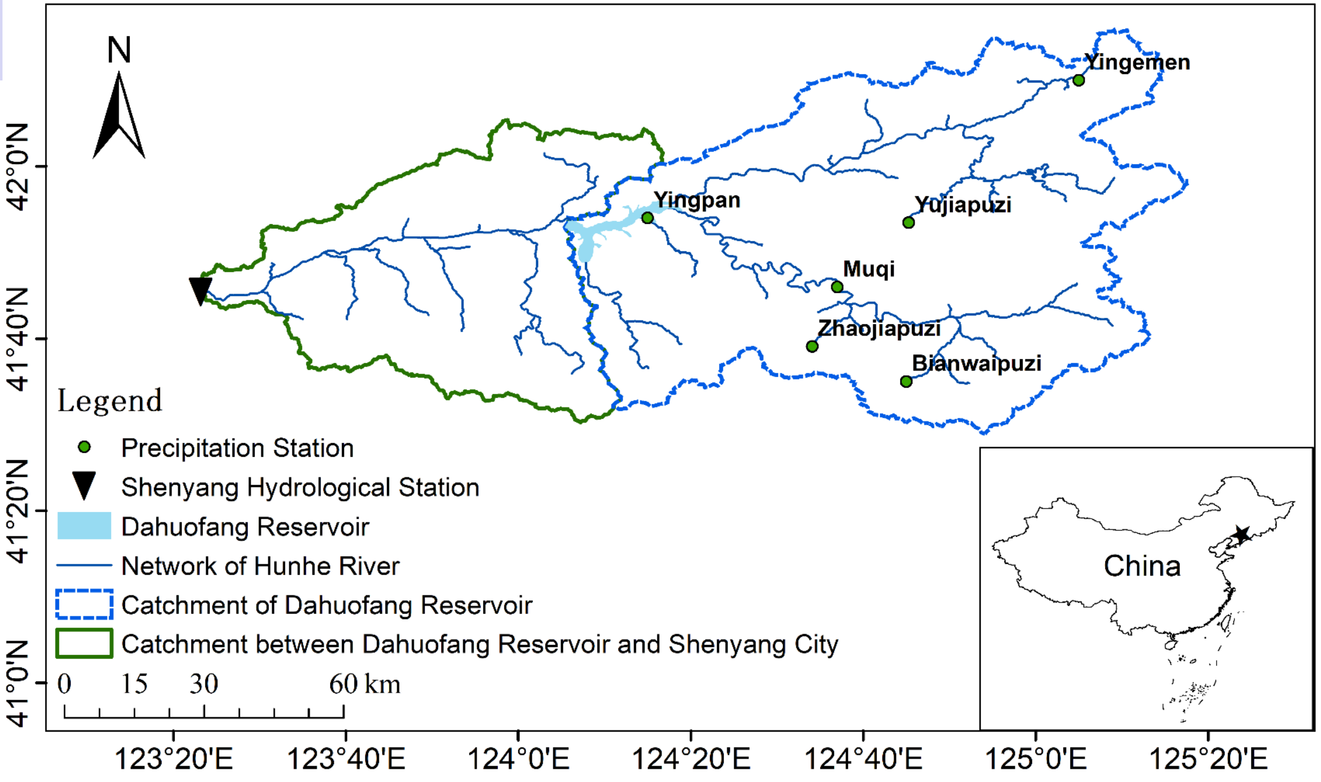


Fig Schematic of Hunhe basin

- **It's necessary to verify the quality of precipitation forecasts and provide reservoir managers with a reliable prediction of risk for flood control in advance.**

3 Data and study area

Observed precipitation & Forecast precipitation

- **Observed precipitation** The observed daily precipitation from **2007-2018** (from **May–October**, which is **flood season**) is collected from the **six precipitation ground stations**.

- **Forecast precipitation** The forecasts of four products were selected in this study: **CMA, ECMWF, NCEP,** and **UKMO**.

- In order to be consistent with the base time of the observed precipitation, we only selected the forecasts with the **base time** at **00:00 UTC**.
- The **lead time** of forecasts for this study is **1, 2, 3, 4, 5, 6, 7 days.**

Table Configurations of the four forecasts products investigated in this study

Products	Time range (Days/Hours)	Resolution (˚)	Base time (UTC)
CMA	d 0-15	0.2815×0.2812	0/12
ECMWF	d 0-15	O640 (ORGG)	0/12
NCEP	d 0-16	1×1	0/6/12/18
UKMO	h 0-174	0.187×0.2815	0/6/12/18

3 Data and study area

Classification of daily precipitation

Based on the classification standard of the meteorological department of China, we divide precipitation into **four categories**.

	L1	L2	L3	L4			

Table1 Classification standard of daily precipitation by CMA

Magnitude	1	2	3	4	5	6	7
Classification standard of precipitation	No rain	Light rain	Medium rain	Heavy rain	Rainstorm	Heavy rainstorm	Extreme rainstorm
Amount of daily precipitation (mm)	0–0.1	0.1–9.9	10.0–24.9	25.0–49.9	50.0–99.9	100.0–249.9	>250.0

Table 2 The number of samples of observed precipitation in Muqi precipitation station from 2007 to 2018 (May to October)

Rainfall Station	No Rain	Light Rain	Medium Rain	Heavy Rain	Rainstorm	Heavy rainstorm	Extreme rainstorm	Sum
Muqi	1444	522	149	56	**22**	**3**	**0**	2196

4 Results

Verification for Accuracy

Observations VS forecasts

- The forecasts of four products are basically **unbiased** in the four lead time.

- **No one** of the four products is **best** or **worst** in all lead time.

- With the increase of lead time, the **quality** of forecasts show **no obvious trend**.

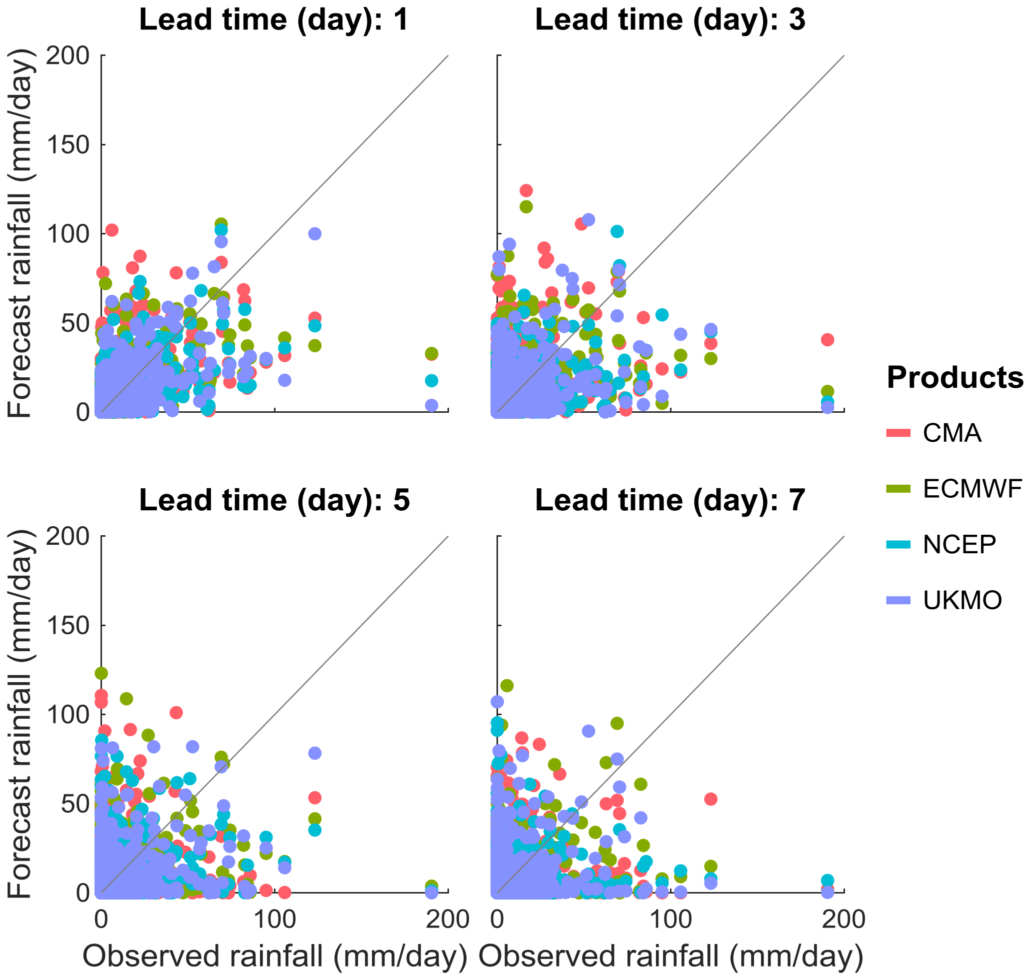


Fig Observations versus +1 day, +3 day, +5 day and +7 day forecasts in Muqi precipitation station.

4 Results

Verification for Accuracy

Box and whisker plots

- The forecasts of four products are basically **unbiased** in the four lead time.

- **No one** of the four products is **best** or **worst** in all lead time.

- With the increase of lead time, the **quality** of forecasts show **no obvious trend**.

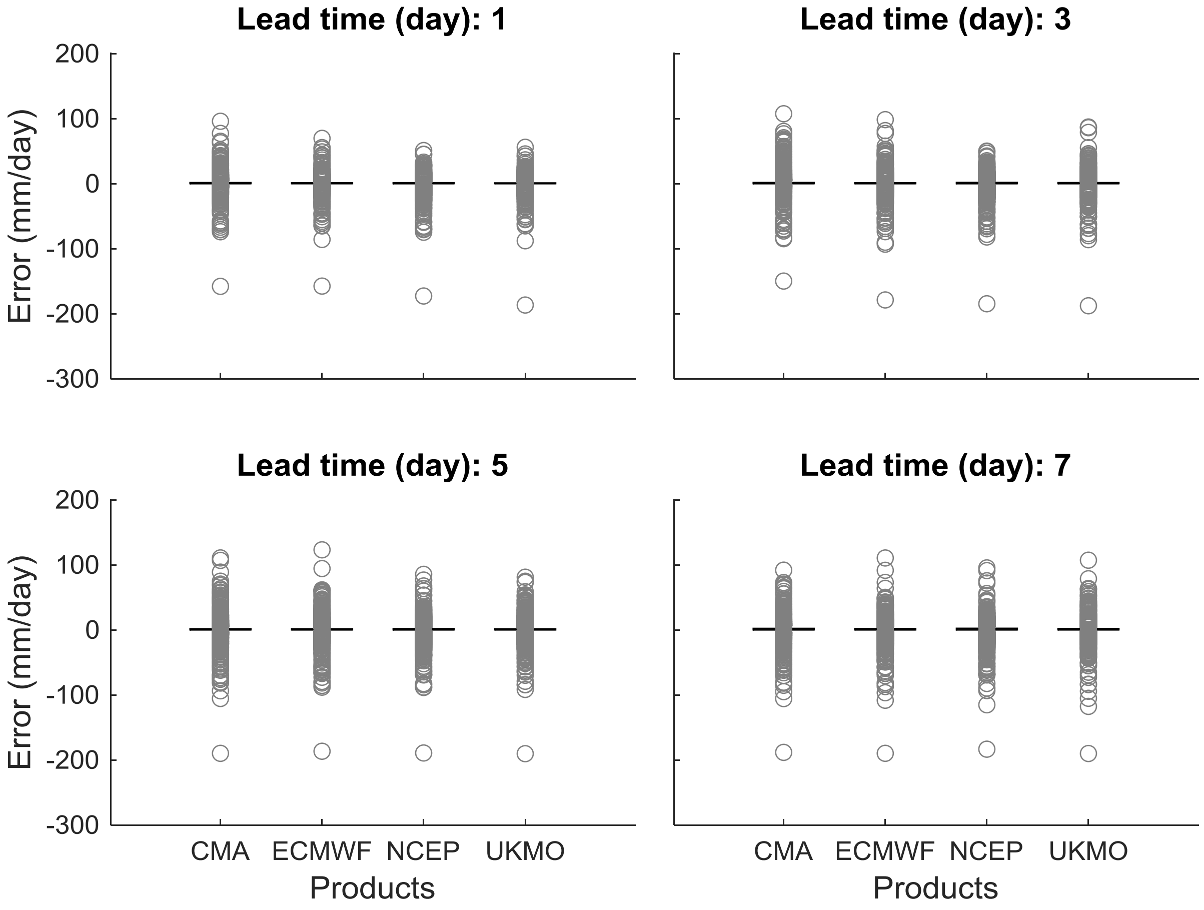


Fig Box and whisker plots of +1 day, +3 day, +5 day and +7 day forecast errors in Muqi precipitation station.

4 Results

Verification for Accuracy

RMSE

- **NCEP** performs **best** and **CMA** performs **worst**.

- The **accuracy** of forecasts **decreases** with the increase of lead time.

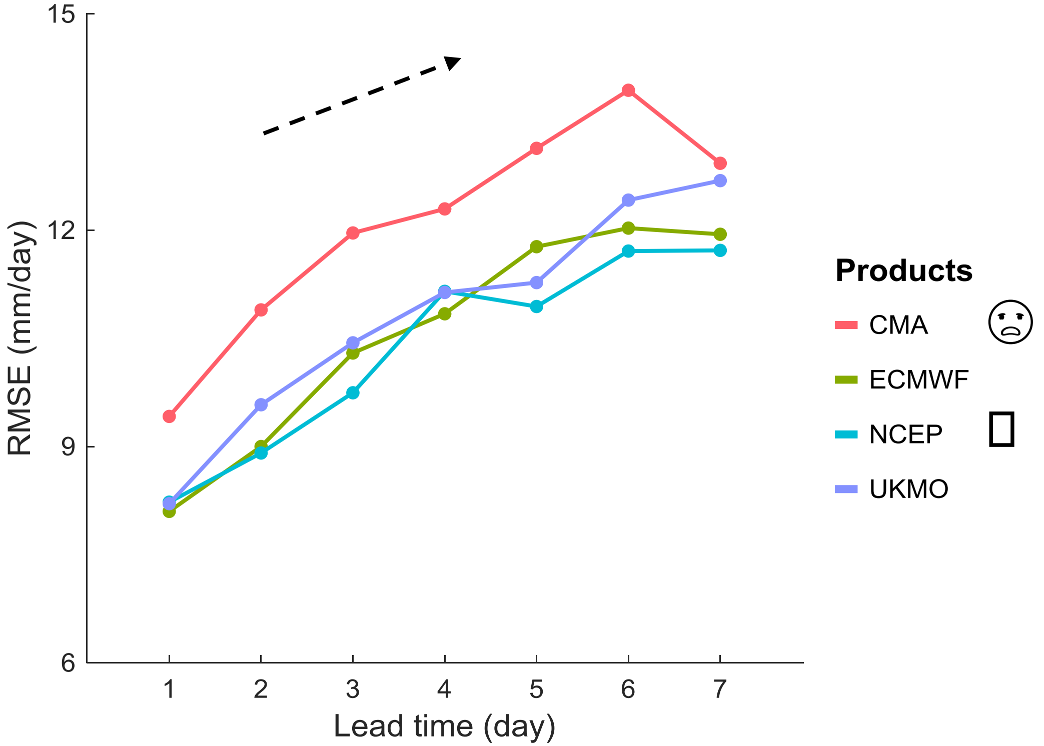


Fig RMSE of the forecasts in Muqi precipitation station.

4 Results

Verification for Comprehensive Uncertainty

NMI

- **ECMWF** performs **best** and **NCEP** performs **worst**.
- The **comprehensive uncertainty increases** with the increase of lead time.

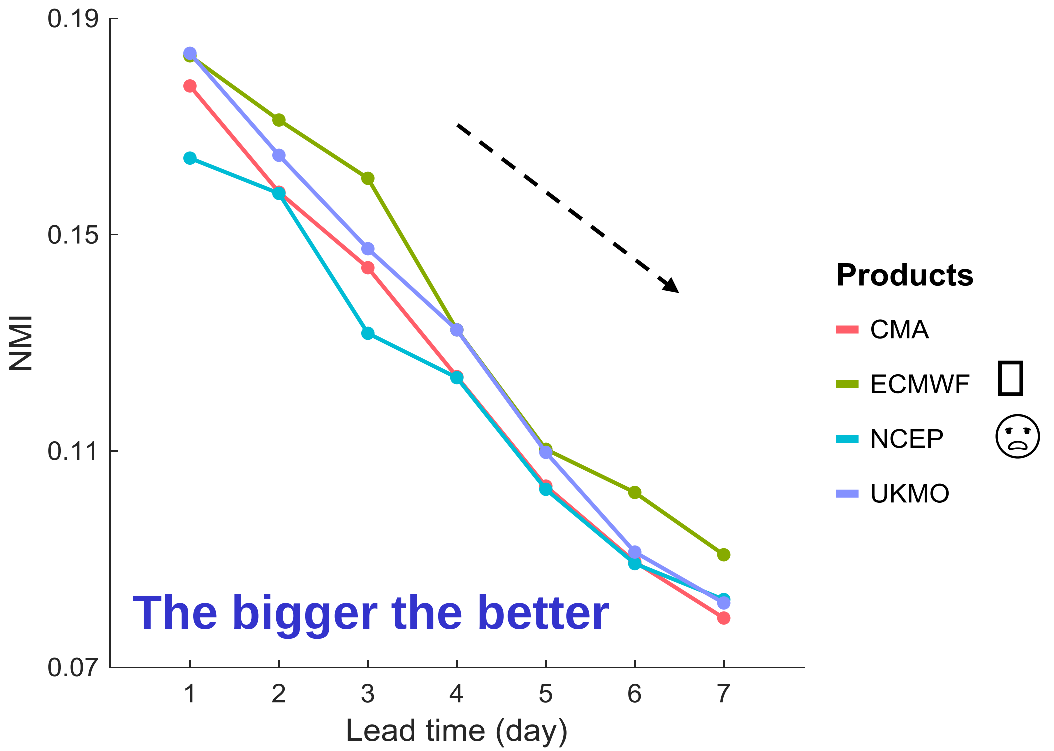


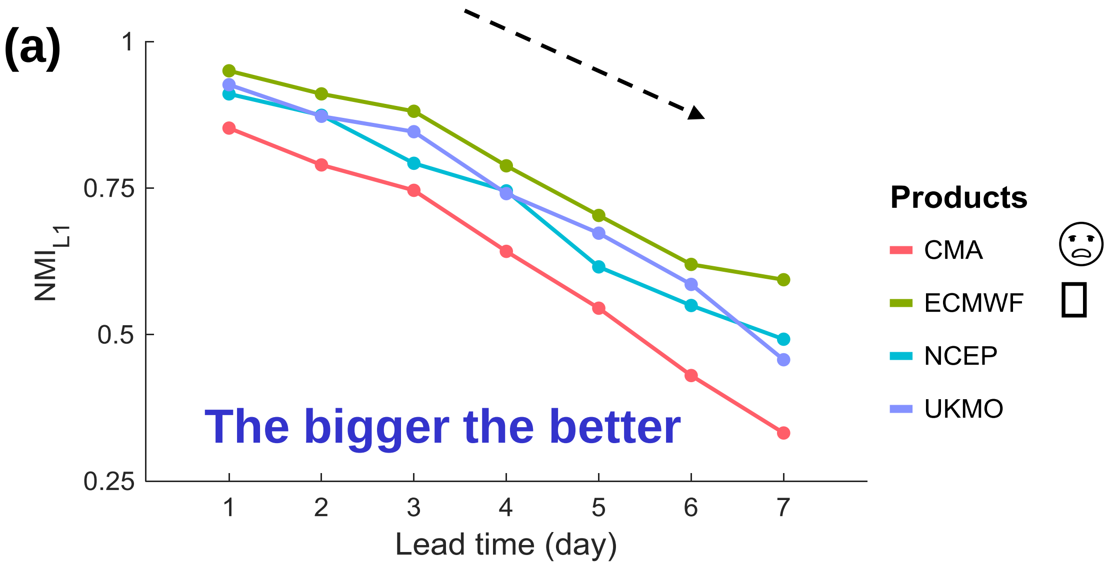
Fig NMI of the forecasts in Muqi precipitation station.

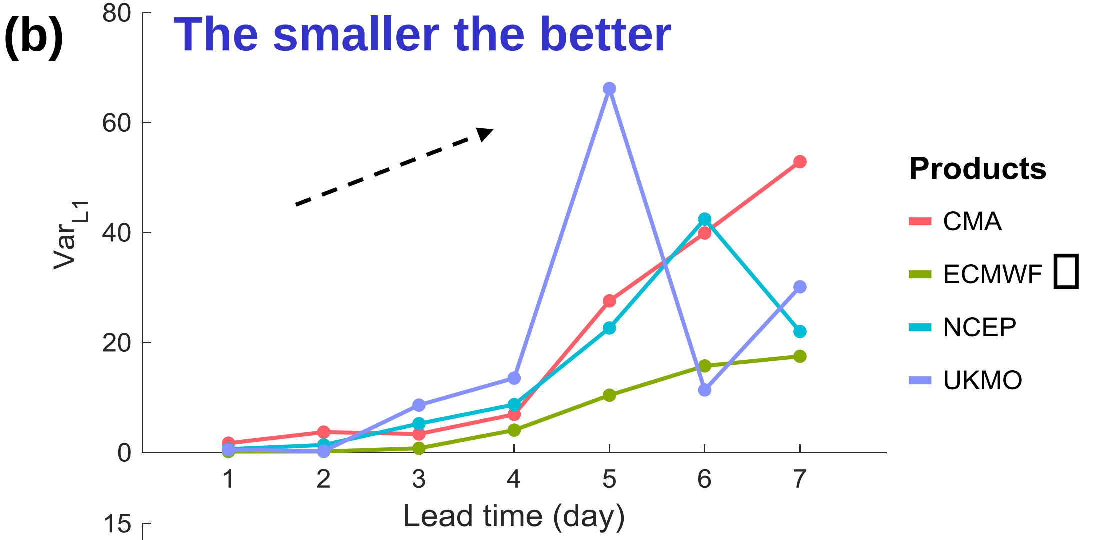
4 Results

Verification for Uncertainty | L1

NMI_{Li} & Var_{Li} & WPI_{Li}

$$Var_{Li} = E\left[\left(X - E(X)\right)^2 | y_{Li}\right] \qquad WPI_{Li} = f_{Li}^{95\%} - f_{Li}^{5\%}$$





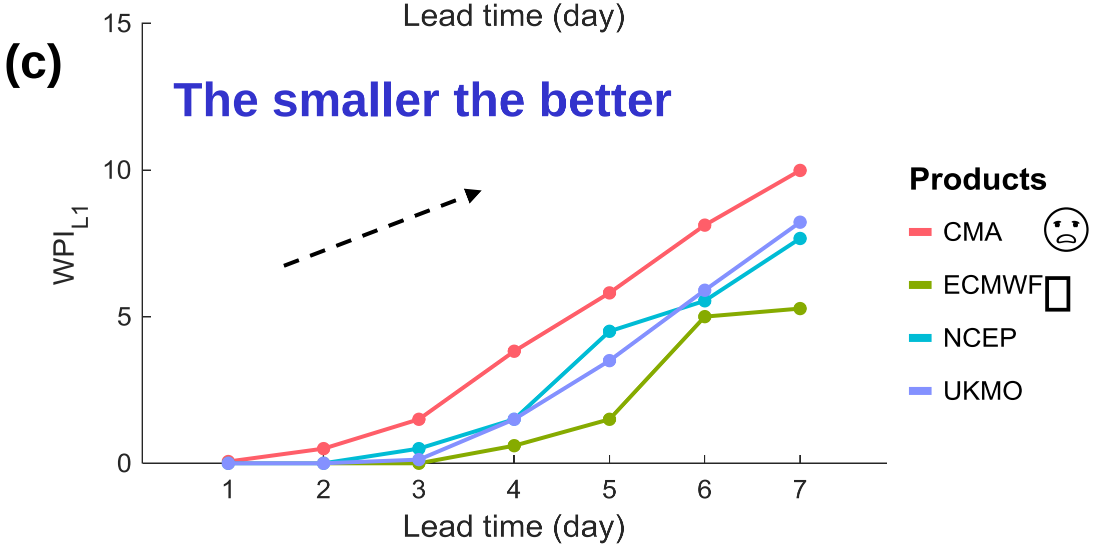
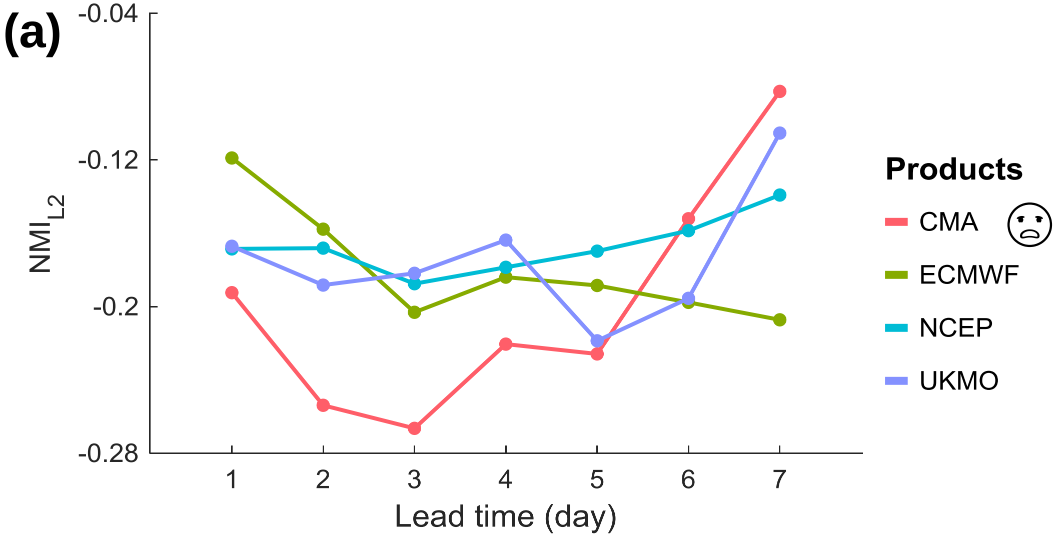


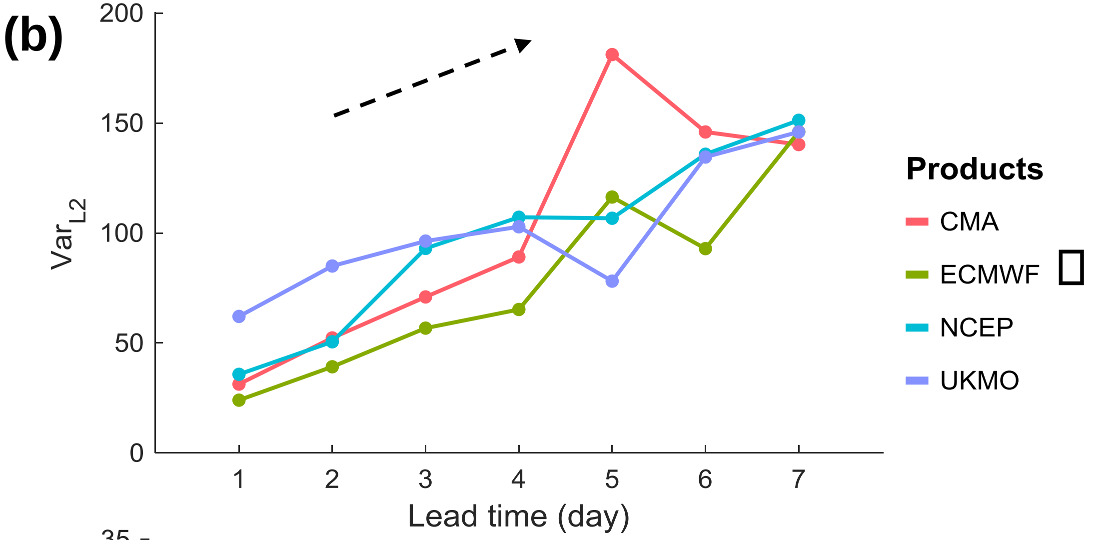
Fig The NMI_{L1} (a), Var_{L1} (b) and WPI_{L1} (c) in Muqi precipitation station.

4 Results

Verification for Uncertainty | L2

NMI_{Li} & Var_{Li} & WPI_{Li}





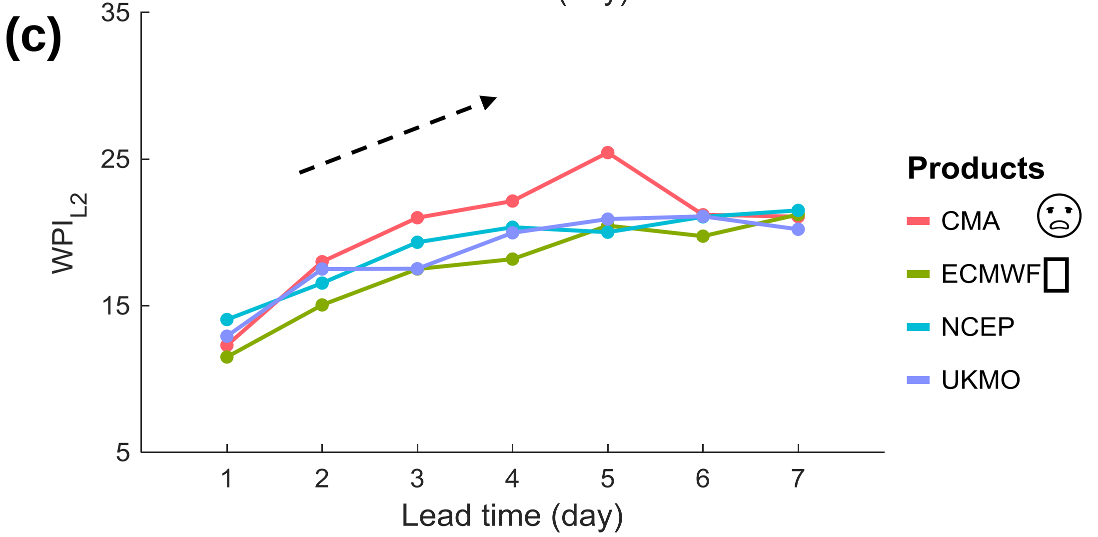


Fig The NMI_{L2} (a), Var_{L2} (b) and WPI_{L2} (c) in Muqi precipitation station.

4 Results

Verification for Uncertainty | L3

NMI_{Li} & Var_{Li} & WPI_{Li}

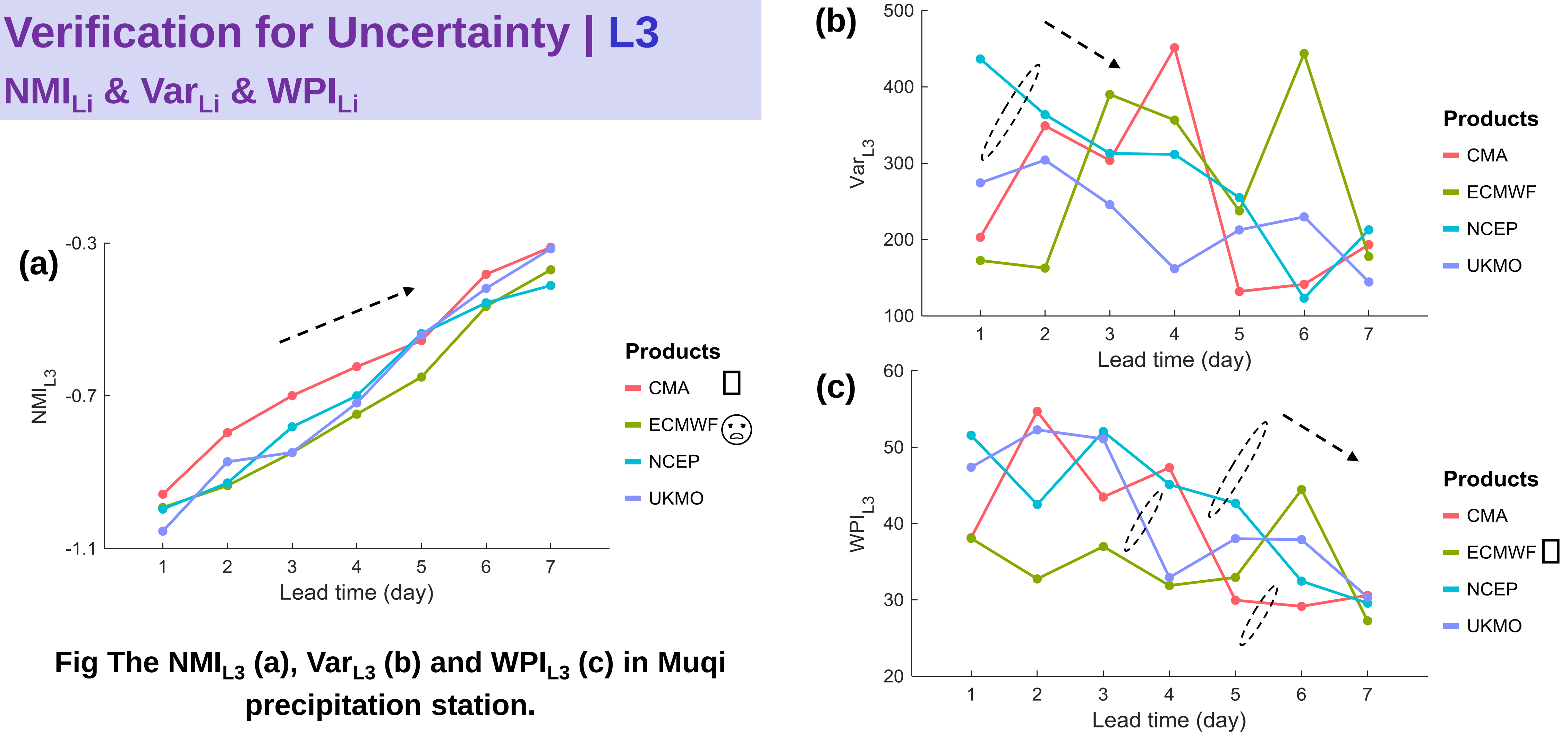
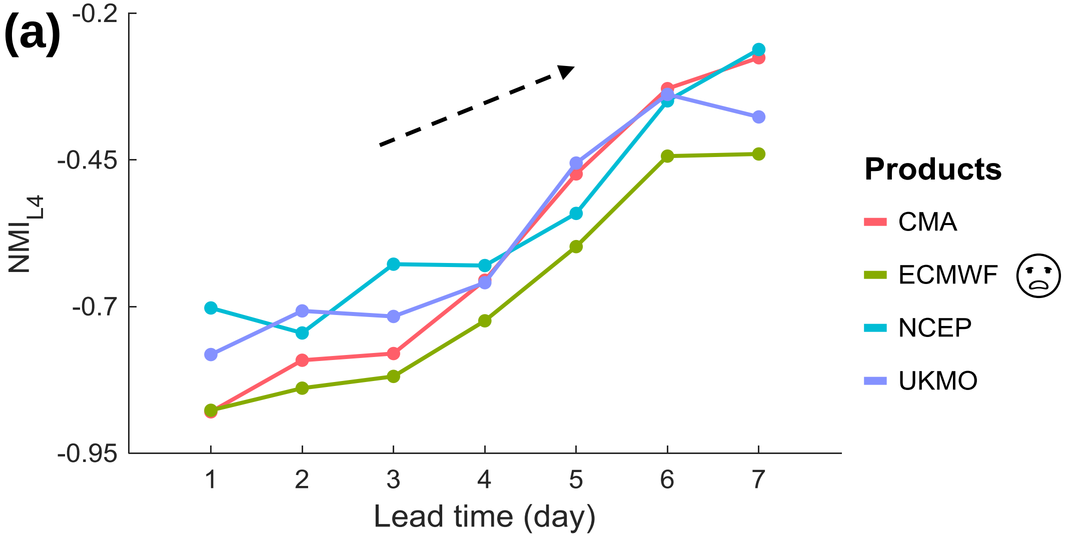


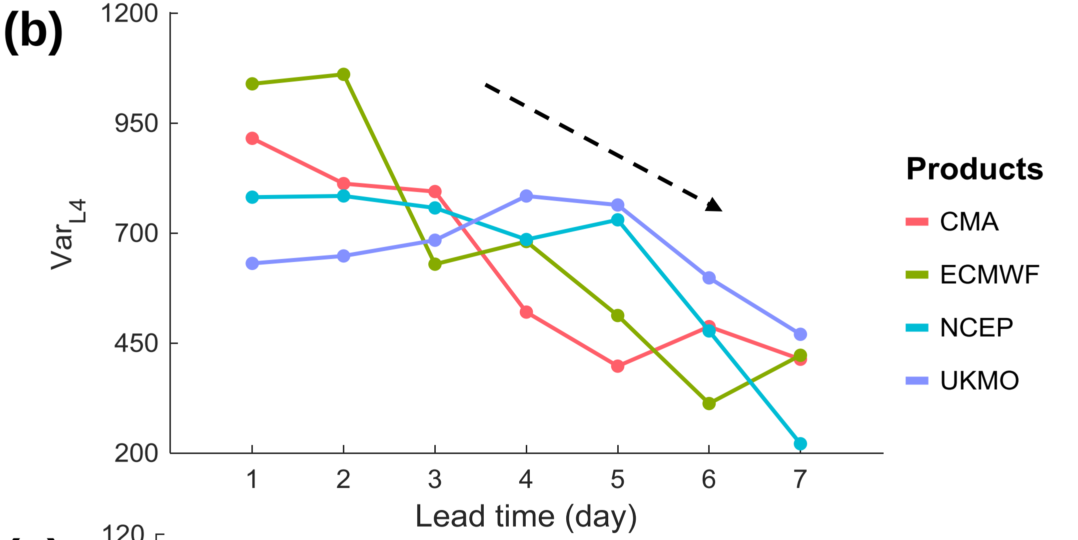
Fig The NMI_{L3} (a), Var_{L3} (b) and WPI_{L3} (c) in Muqi precipitation station.

4 Results

Verification for Uncertainty | L4

NMI_{Li} & Var_{Li} & WPI_{Li}





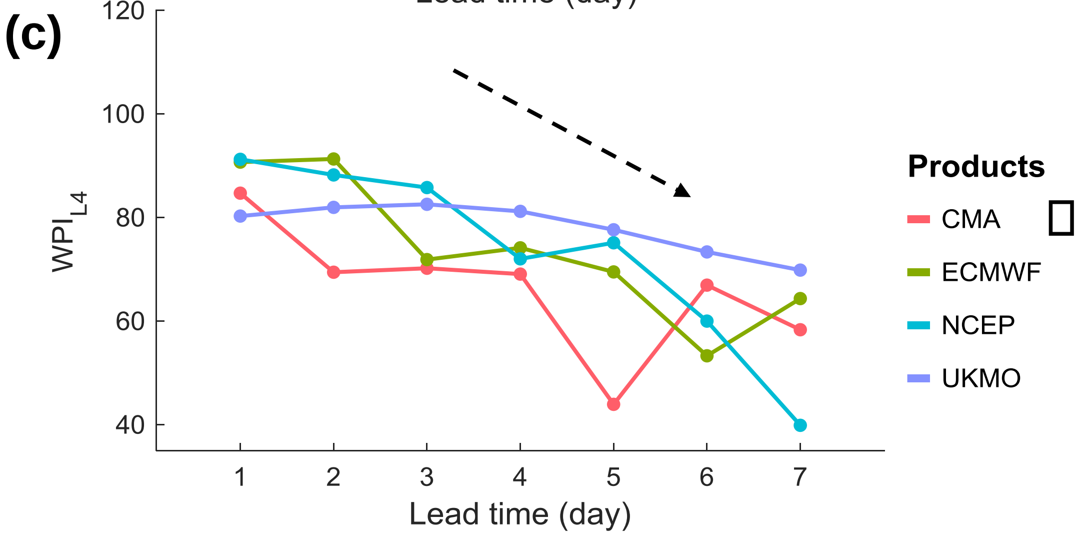


Fig The NMI_{L4} (a), Var_{L4} (b) and WPI_{L4} (c) in Muqi precipitation station.

5 Conclusions

- There exists **no forecast product** which performs **best or worst** by **all selected verification methods**.

NMI

- The **comprehensive uncertainty** of all categories of four products **increases** with the increase of lead time.

NMI_{Li} & Var_{Li} & WPI_{Li}

- The **uncertainty** basically shows an **increasing** trend with the increase of lead time in **L1 and L2** categories. However, the uncertainty of some forecast products in **L3** category and all four products in **L4** category **decreases** with the increase of lead time.

- NMI_{Li} is **effective** for **distinguishing** the **performance** of different forecast products and **investigating** the **trend** of forecasts performance with the increase of lead time,.

Thanks!

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